

/* Models for Seasonal Time-Series Data */

→ Seasonal ARIMAs

[Seasonal Autoregressive Model: ARMA(1,0)s]

$$y_t - \mu = \phi_s (y_{t-s} - \mu) + \epsilon_t \quad \text{or} \\ (1 - \phi_s B^s)(y_t - \mu) = \epsilon_t$$

[Seasonal Moving Average Model: ARMA(0,1)s]

$$y_t = \theta_0 + (1 - \theta_s B^s) \epsilon_t$$

[Seasonal Autoregressive Moving Average Model: ARMA(1,1)s]

$$(1 - \phi_s B^s)(y_t - \mu) = (1 - \theta_s B^s) \epsilon_t$$

[Seasonal Multiplicative Autoregressive Integrated Moving Average Model]

→ ARIMA(p,d,q)(P,D,Q)s

(p,d,q) : Nonseasonal part of the model

(P,D,Q)s : Seasonal part of the model

cf) ARIMA(1,1,1)(1,1,1)⁴

$$(1 - \phi_1 B)(1 - \phi_1 B^4)(1 - B)(1 - B^4)y_t = (1 - \theta_1 B)(1 - \theta_1 B^4)e_t$$

① ② ③ ④ ⑤ ⑥

① nonseasonal AR(1)

② seasonal AR(1)

③ nonseasonal difference

④ seasonal difference

⑤ nonseasonal MA(1)

⑥ seasonal MA(1)